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# IS COGNITIVE STYLE RELATED TO LINK BETWEEN PROCEDURAL AND CONCEPTUAL MATHEMATICAL KNOWLEDGE?

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**Abstract.** This study examined the relation between cognitive style and link between procedural and conceptual mathematical knowledge. It used a sample of 34 mathematically talented eleventh-grade students. A significant positive correlation was found between the students' achievements on the administered Embedded Figures Tests (where "field-dependence-independence" cognitive style has a very specific perceptual connotation) and the measures of link between their scores on procedural and conceptual mathematical knowledge. The same relation was again found in a group of particularly talented students who participated in mathematical competitions ($N = 16$), but not in the control group comprising other talented students ($N = 18$).

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*Key words and phrases*: Cognitive Style, Procedural Knowledge, Conceptual Knowledge, Linking Procedural and Conceptual Knowledge, Talented Students, Mathematics Education, Upper Secondary Education.

## Introduction

As regards cognitive style, academically gifted students may be more field independent than their counterparts involved in the regular education program [1]. A previous analysis of the structure of test achievements (including EFT test [2]) shows that highly gifted students in mathematics and technical sciences, who are scholarship holder candidates in Serbia, are characterized by a form of general fluid intelligence contained in figurative tests [3].

Linking procedural and conceptual mathematical knowledge is an important yet neglected goal of mathematics education, the attainment of which is a complex but achievable enterprise [4]. When the effects of developing procedural and conceptual knowledge and establishing links between them are examined, cognitive style should be included since some students, because of their less flexible (say more field dependent) cognitive style, may demonstrate unbalanced gains in these knowledge types resulting in missing or poor links between them [5].

Having in mind the presented research context, the objective of this study was to examine the relation between cognitive style and link between procedural and

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conceptual mathematical knowledge. The rest of this paper presents how these variables were operationalized, what were the main results, and why such results might be obtained.

## Method

The study used a sample of 34 mathematically talented students who came from two eleventh-grade classes of "Matematička Gimnazija"—the specialized high school in Belgrade for mathematically talented students.

The study had a correlative design. The variables were: class (1 - control group comprising self-financed students, 2 - target group comprising students financed by the state; all these students passed the school entrance examination, but free education is reserved for those who achieved better total scores at that examination), cognitive style, procedural knowledge, conceptual knowledge, and link between procedural and conceptual knowledge (hereafter denoted by P-C link).

Cognitive style was measured by the last (perhaps the hardest) 16-item subtest of Bukvić's modification of Embedded Figures Tests [2] standardized for Yugoslav population. This instrument was administered under a group setting (one class at a time; both classes within 45 minutes) in exactly 10 minutes by a psychologist (the second author of this report). The subject's cognitive style was represented by the first principal component factor score obtained from the subjects' answers. The factor score reliability (Lord-Kaiser-Caffrey) was .83.

Procedural and conceptual knowledge were measured by scores given to different solutions of the following task taken from [6]: "In the square below, $M$ and $N$ are midpoints of the corresponding sides. Determine the numerical value of sin."

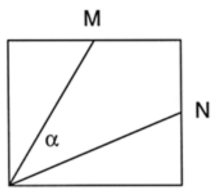


Fig. 1

Answers to this task were written in a questionnaire administered under a group setting (one class at a time; both classes within 45 minutes) in exactly 20 minutes by a mathematician (the first author of this report) who also precisely scored the students' answers. For each correct solution, student received 1 point for conceptual knowledge and 1 point for procedural knowledge. Partial credit was given when: (1) student wrote a solution plan (how the task can be solved) that was partially or fully correct (for conceptual knowledge), and (2) some of the required calculations (plan implementation) were performed correctly (for procedural knowledge).

P-C link was measured by formula $2PC/(P^2 + C^2)$ introduced in [7], where $P$ and $C$ denote total scores on procedural and conceptual knowledge, respectively. For those students where $PC$ equaled 0 (when one or both types of knowledge was (were) not demonstrated), P-C links were equal to 0. Note that such a defined link takes values from interval $[0, 1]$, where a bigger number indicates a stronger P-C link.

The data collection was realized at the end of the fall semester in January 2004 during regular school lessons. The authors told the subjects that this study would examine their problem solving performance and the subjects willingly provided the requested data.

## Results

The correlations among procedural knowledge, conceptual knowledge, P-C link, cognitive style, and class are presented in Table 1.

Table 1. Correlations among the examined variables

| VARIABLE | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1. procedural knowledge | .82** | .74** | .25 | -.11 |
| 2. conceptual knowledge | | .56** | .24 | -.03 |
| 3. P-C link | | | .34* | -.38* |
| 4. cognitive style | | | | .19 |
| 5. class | | | | |

$^{*}p < .05$, $^{**}p < .01$

The correlations between cognitive style and P-C link for the two classes are given in Table 2.

Table 2. Correlations between P-C link and cognitive style for the two classes

| CLASS | CORR |
|---|---|
| 1. control group of talented students | .31 |
| 2. particularly talented students | .55* |

$^{*}p < .05$

## Discussion

Two important findings emerged from this study.

First, there was a significant positive correlation between cognitive style and P-C link.

Second, while this relation also held true for the target group of particularly talented students participating in mathematical competitions, this was not the case for the subjects of the control group.

As Table 1 evidences, there were no significant differences between the two classes with respect to cognitive style, procedural knowledge and conceptual knowledge, which is acceptable (not expectable) as all these students belong to the same highly selected student sample (two students solved the problem in 3 ways and two in two ways; three students pursued a specific way of solving the problem—not listed among 16 different solutions summarized in [6]—and one of them succeeded;

an easier variant of this solution is given in the appendix). But, while particularly talented students were competition oriented (even in 20 minutes, two students solved the problem in 3 ways), most students in the control group were not so directed. So, the competitors, who obtained more unbalanced scores on procedural and conceptual knowledge than those in the control class (recall that correlation was $-.38$), may in general, compared to other talented students, be more prone to procedural errors or calculation ignorance when conceptual knowledge is correct, which was evidenced by the student's answers (P-C link for 8 students, a half of the group, was 0; in the control group just 3 students had such a link). However, the more competitor's cognitive style was field independent, the stronger P-C link he/she established, which is an important finding that, to our reading, has not been reported so far. This was, however, not the case for those in the control group, which, among others, might be caused by a less competitive approach to day-to-day learning requiring a more relaxed cognitive processing. Another reason was suggested by the examined data: while no significant differences between the classes were found for the variance of the link measure (.20 vs. .11; Levene's Test: $F = 2.90$, $p = .098$), the variance of the cognitive style measure was higher in the competitor class (1.43 vs. .61; Levene's Test: $F = 6.81$, $p = .014$), which was a less homogenous sub-sample. Despite such plausible explanations, further investigations are still needed, which should also include ordinary (not mathematically talented students) from gymnasium or vocational school.

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## Appendix—A Specific way of solving the problem

As the hypotenuses of the shaded right-angled triangles (Fig. 2) are perpendicular (one triangle can be rotated into the other by angle of 90°), $AD$ is an altitude of triangle $ABM$. If $AB = 2$, the area of triangle $ABM$ (or $BMA$) is 2, and, since $BM = \sqrt{5}$, $AD$ is equal to $\frac{4}{\sqrt{5}}$. As $\cos\alpha = \frac{AD}{AM} = \frac{4}{5}$, the numerical value of $\sin\alpha$ is $\frac{3}{5}$(obtained from the relation $\sin\alpha = \sqrt{1-\cos^2\alpha}$). The above-mentioned successful solver utilized the fact that $MD$ is an altitude of triangle $ANM$, whose area, compared to that of triangle $ABM$, cannot be obtained at a glance.

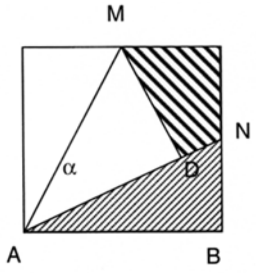

Fig. 2

However, when the length of $MD$ is known, the numerical value of $\sin\alpha$ can easily be found (from the relation $\sin\alpha = \frac{MD}{AM}$).

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