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THE INSTRUCTION OF EUCLIDEAN GEOMETRY WITH THE USE OF EDUCATIONAL SOFTWARE IN MULTILINGUAL CLASSES

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Abstract. In this article we aim to describe how the use of computer visualization software can assist the teaching of Euclidean geometry in a multilingual classroom. In particular, the difficulties that the students may experience are presented, together with some advantages and disadvantages that the use of such educational software may have. We finally conclude with examples demonstrating the use of these techniques.

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1. Introduction

It is widely acknowledged that language plays an important role in the teaching and learning of mathematics (e.g., Laborde 1990). In the field of Euclidean geometry in particular this significance is even greater because of the wide range of the mathematical glossary used (a variety of logograms and pictograms) and the increased use of mathematical discourse in the proofs of geometrical theorems (geometry requires the use of elaborate language statements more than other fields, such as algebra where the mathematical proofs require more symbolic notations). In this article, we aim to enhance the teaching of Euclidean geometry by using educational software. This software constitutes an extremely fruitful educational object in multilingual classes, since it helps the students to understand geometrical theorems by visualization, making it ideal for situations where not all the students have the same level of knowledge of the spoken language. Moreover, since it has been known that the mother tongue of a student and the grammar structures that the language uses affects the cognitive development of the student (Bell 1998), the use of computer tools to assist the instruction of geometry helps to eliminate these differences among the students.

In this paper we propose the methodology for the introduction of such a method which is composed in two main stages:

- In the first stage the main concepts and symbols of Euclidean Geometry are introduced in the form of a "Geometrical Glossary", concentrating in the visual understanding of the underlying concepts with emphasis in pictorial geometry.
- In the second stage, we propose a series of computer software scenarios (written in the computer language Java) which, using only the geometrical glossary, formulate geometric theorems and through of the activities that follow from the interaction with software, lead the students to understand and potentially apply them further.

We thoroughly analyse:

1. The educational techniques
2. The educational aims
3. The technical requirements
4. The requirements of the teachers (extremely high) and those of the students (the least possible)
5. The overall evaluation of the educational work.

The main aim of the presentation is to familiarise multilingual classes to the greatest possible extent with Euclidean Geometry, through a student-oriented tuition, characterised by activities that are supported by suitable software.

2. The difficulties of students in multilingual classes during the instruction of Euclidean Geometry

- The insufficient knowledge of the Greek language leads to difficulties in the understanding and the visual representation of geometrical concepts.
- The language used by the class teacher during the instruction of Euclidean Geometry is a mixture of the every day language used by the students together with the language (terminology) of Geometry. Hence, both these factors create a big problem in the communication of the students of multilingual classes concerning the understanding of the new information that is being transmitted to them.
- In almost every sentence, the teacher of Geometry uses terms that do not exist in everyday conversation or, if they exist, have a different meaning. Among these terms, very few will be used later and some of them will be heard very rarely or never again. As a consequence, the students will deteriorate their yet poor linguistic ability.
- Many problems are also derived from the differences that exist in the conventions used in Mathematics and every day speech. On the one hand, teachers employ the geometrical terminology in a strict way, whereas on the other hand the students usually interpret their speech according the lines of every day linguistic conventions, yielding significant problems in perception.
- Students must spend much effort in understanding the language of Geometry in the classroom and in successfully expressing with concision and precision the necessary geometrical concepts in a language that is almost unknown to them and moreover these concepts are inherently difficult to fathom due to their theoretical character (being mathematical).
- Most of the geometrical concepts are used, just like most students, without any significant effort and comprehension of the significance of the geometric terms.
- The visual aids are most of the times used insufficiently or wrongly.
- There is a difficulty due to the fact that no visual aid can fully substitute a geometrical concept.

- The teacher must employ a certain vocabulary and not to use synonym words for the same concept.

The usual method of teaching Euclidean Geometry is expanded as follows:

In the beginning, the verbal representation of a theorem (or proposition or corollary or exercise) is presented, which the student is asked to visually implement (a figure) and solve using the interpretation of its geometrical properties. It is obvious that this method presents problems in multilingual classrooms, and for this reason in the present work a different methodology is presented, which also combines the development of different abilities by the students.

3. Euclidean Geometry and Educational Software: advantages and disadvantages

In an attempt to develop new teaching techniques in multilingual classes, the new technologies constitute an important assistant. Specially selected software can substantially help the students in the full understanding of even the most difficult concepts and mathematical propositions.

The selection of the topics supported by the educational software in this paper relates to planar geometry up to the level of metric relations, where motion is an important factor in the comprehension of the geometric properties.

A suitable computer environment can help the investigation of the various questions that may arise form a certain problem, enabling:

a. The pictorial representation of many possibilities/cases on the screen
b. The dynamic processing and adjustment of these representations by the user. In that way, the user can create new assumptions, reflections, extensions and even new theorems via a dynamic investigation of traditional theorems (Raptis, p. 163).

Nevertheless, it should be stressed that the education software, despite its contribution to immediate understanding, very difficult entices the students to the procedure and self-action. The development of educational software of various cognitive levels with an endeavour of selective introduction of the students to the probative process would be extremely interesting.

4. Educational Methodology

The methodology that follows, is indicatively presented through one specific application of educational software, namely the barycentre (centre of mass) of a triangle. With exactly the same structure almost all the propositions of Plane Geometry can be presented.

Glossary:

(a) A few basic points a teacher of mathematics should bear in mind when planning to teach a particular section are mentioned below:
    - Every teacher should be aware of the verbal problems that the teaching of geometry creates.

- They should create better geometric perceptions using a greater variety of figures and visual aids.
- They should employ the geometric terminology with clarity and precision.
- They should reduce the number of terms used to the absolute minimum.
- The terminology should not be an end in itself during the geometrical instruction but should be a tool aiming in the expression of geometric concepts.
- The students should express themselves in the classroom giving the opportunity to themselves and also to the teacher to understand whether the 'term' words used are common to both of them or not.
- The communication problems should be solved in common.
- The linguistic communication problems should be solved in the classroom.

For the presentation of this particular activity, the following geometrical 'glossary' is necessary.

The students should know:

- How to construct any triangle (equilateral, scalene, acute, right, obtuse).
- What a median of a triangle is and how many medians there are.
- How to construct the median of a triangle using a rule and compasses.
- That the line segment joining the midpoints of two sides of the triangle is parallel the third side and is equal in length to a half of that side.
- That, if a quadrangle has two sides equal and parallel to each other then it is a parallelogram.
- That the diagonals of a parallelogram are bisected by their intersection point.

(b) Educational Method

The method that we are trying to implement is a guided instruction technique with elements of active discovery and the use of a computer. A main goal of the educational method is the motivation of the student's interest and curiosity, which can be obtained as follows:

- The teacher with frequent questions, which relate mainly to existing knowledge—the necessary 'glossary' in each case—tries to stimulate the interest and the curiosity of the students leading to their active participation during the teaching.
- The following figures (in that order) are displayed on the computer screen and the students are asked to formulate the corresponding propositions:

(i)

$$MN =\parallel \frac{BC}{2}$$

(ii)

If $AB =\parallel CD$ or $AD =\parallel BC$

(iii)

If $ABCD$ is a parallegoram then
$OA = OC$, $OB = OD$.

(iv)

Median:
The line segment that joins one vertex of a triangle with the midpoint of the opposite side, is called a median.

(v) To stimulate the interest of the students even more, the teacher may present the terminology of the 'median' using the most representing, for the majority of students, exercises.

Next, the teacher defines the medians m_a, m_b, m_c that correspond to the sides a, b, c.

(vi) The construction of the median of a triangle using a straight rule and compasses is displayed on the computer screen.

(c) Direction of the instruction of the course

The teacher asks the students:

- To draw in their notebook a scalene triangle ABC and draw the medians AM and BN that intersect at G.
- To draw the line segment NM and to discover its relation with AB.
- To take the midpoint K of AG and the midpoint L of BG and to discover the relation between KL and AB.
- To work on the rectangle $KLMN$.

- To write down the relations between the line segments AG, AM, GM and BG, BN, GN.
- To formulate similar results about the third median CO.
- It should be mentioned by the teacher or a student with good knowledge in Physics that the point of intersection of the medians is called the barycentre or centre of gravity of the triangle.
- The physical significance of the centre of gravity should be stated.

- Various triangles and their corresponding centres of gravity should be presented and commented on.

(d) Educational aims

- The students should understand that in any triangle the medians pass through a common point.
- They should know that this point of intersection is called the barycentre or centre of gravity of the triangle and realise its significance in physics.
- They should learn the relation between the median, the distance of the barycentre to the vertices of the midpoint of the corresponding edge (relation of the three line segments).

(e) Technical requirements

A computer for use in the classroom that is able to use the Java computer language. The advantage of using Java over other similar computer languages is that it is extremely easy to translate geometrical theorems into that language and, moreover, Java applications can be easily placed on an internet web page, therefore making it more accessible to the students.

(f) Evaluation of the educational work

An essential requirement for the smooth transition from one educational topic to another is the evaluation of the educational work.

A proposed scheme for the evaluation of this particular application is the following: The teacher asks from a student

- To formulate the theorem again.
- To sketch the main points in the proof of the theorem.
- To answer why the barycentre is always in the interior of the triangle.
- To prove that the triangle with vertices the midpoints of the sides of a triangle ABC has the same barycentre as ABC.

5. Extensions—Conclusions

Summarising the educational method described in the previous section, we observe that this method is somehow the 'inverse' of the usual.

Initially, we present the necessary knowledge, then we present the figure—without its verbal representation—and through the motion the student is progressively guided to the understanding of the particular geometric properties. Finally, the statement of the theorem is given, where any verbal difficulties are tackled through the already obtained geometrical understanding.

The probative process still remains at a high level of difficulty and is simply *assisted* by the new technologies.

The more general development of an educational software that includes sections of Metric Geometry as well as Solid Geometry and which will incorporate the probative process—whenever possible—is already an object of study by numerous researchers in the field of the didactics of Mathematics.

6. Appendix

We present here some outputs of an implementation of the above ideas by a small demonstration software. The advantage Java or Web-based is that users can use their computer mouse and modify the shape of the triangle and see how the different characteristics of the theorem change. In the following figures, the Euler 9-point theorem is described, which states that certain nine points in every triangle should lie on the same circle. The student can then change the shape of the triangle by moving its vertices and verify that, no matter how the triangle looks like, these nine points will always be on a circle.

Fig. 1. A Java demonstration of the Euler 9-point theorem

Fig. 2. The user can move the vertices and see who the various points change

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