THE TEACHING OF MATHEMATICS
2009, Vol. XII, 2, pp. 83–102

# STUDENT PLACEMENT IN CALCULUS COURSES. A CASE STUDY

**Radoslav M. Dimitrić**

**Abstract.** This is a continuation of the author's study on making and using course specific diagnostic tests in college setting. First a predictive power of SAT scores on student performance in calculus classes is discussed and it is shown that SAT scores cannot be used as a useful predictor of students' performance in those classes. Lack of usefulness of SAT scores as a reliable predictor of students' performance has been noted by other authors as well. Reasons for the lack of relevance are given. The necessity of a "home-made" diagnostic test resulted in creation of such tests and one such test is completely exhibited here. This test proves to be much more useful in gaging students' future performance, to the extent that it would be unrealistic to expect greater explanatory power of students' subsequent performance in class. Uses of the test and the resulting statistics are discussed; comparison with a similar test for statistic classes is given.

*ZDM Subject Classification*: D65; *AMS Subject Classification*: 97D70, 97I40.

*Key words and phrases*: SAT; diagnostic testing; student placement; assessment; predictions of students' performance; Bloom's taxonomy.

Every semester a (college) instructor gets a group of new students that he has to teach, in the first calculus course (in this study). He can find no useful information about his new students on the rosters he holds in his hands. If he goes out of his way and is in favor of the administrators at the college, he may be privileged enough to get some information regarding students' academic background before they came to college: Their school GPA's, their test scores before they came to college or their eventual test scores at the college they just enrolled in.

It has been plentifully noted that it is in nobody's interest to admit an applicant unprepared to succeed in college, thus any device that helps to predict success has clear value both in the admissions process (Barro [2]; McWhorter [11]; Camara [4]) as well in placement of students in appropriate classes.

The instructor (and this author in particular) wants to know what academic endowment his students come with, he wants to know whether his new students are prepared for the class they are about to take, what deficiencies do they have in their knowledge and intellectual abilities, their strengths and potentials, what operational, present abilities do they have and which of them are latent and need to be awakened. Can some (numerical) parameters be arrived at that would measure

A preliminary version of this paper was presented at the Annual joint AMS/MAA meeting in San Diego, on January 7, 2008. The author was scheduled to present this paper at ICME11 in Monterey, Mexico on July 8, 2008, but was unable to attend.

students' likelihood of success (or failure)? And what information and features of the student handling in his institution can he rely on to be able to form a crystal ball that would be fairly reliable in answering his momentous questions?

Placement of students in classes at the proper level and content is a single most important factor in their success in those courses; complementally, improper placement of students results in failures and frustrations both on the side of students and faculty (and inevitably the administrators).

There are different ways colleges decide to place students into (first) calculus courses: The decision is left to the students, College places them on the basis of their *Scholastic Aptitude Test* SAT (also known as SAT I) score(s), SAT scores are used in conjunction with other factors (grades, previous courses ... ), college/department entrance testing (often compound with SAT), after taking pre-calculus and similar courses at the college, some other test like the *American College Test* (ACT) and/or combinations of factors are used.

## 1. SAT (or SAT I)

This is perhaps the most well known test that the graduating high schoolers take (another ubiquitous test being the ACT); it is made and administered by the College Board (their web site: www.sat.org). Its content was somewhat revised in March 2005 to include some material from math in a typical third year of high school; quantitative comparison questions were eliminated. These changes are not essential and not critical to our study since the classes we observe here predate 2005. Scores of the new SAT are adjusted so as to comply with the old test. The SAT tests students in "critical reading," writing and mathematics. Maximum score is 2400. I will only concentrate on the mathematics portion of the test where the range of possible scores is from 200 (minimum) to 800 (maximum) points. The math portion consists of two 25 minute sections and one 20 minute section. One section contains only multiple choice questions. There are a small number of questions where student has to produce an answer that must be a non-negative integer up to 4 digits (grid-ins). The SAT also includes a variable section in critical reading, writing or math (25 minutes) which does not count towards the score, but the student does not know which of the sections is variable. Mathematics questions are in 4 categories: Numbers and operations, algebra and functions, geometry and measurement, data analysis, statistics, probability. There are no questions in trigonometry, radian measure or any formal proofs; planar geometry and three-dimensional geometry are on the pre 8th grade level. The makers of test label about 80% of math questions as "easy" or "medium" the remainder as "hard" (Fox et al [8]). My analysis of typical exams given in (Fox et al [8]) shows that the material on the tests consists of 75% of questions that are at the level of first 8 grades and the reminder covers grades 9-12 (mostly 9-10).

Just what influences performance on the SAT is the subject of many papers. One of them is by (Hawkins [10]) where (the composite) SAT scores are linearly predicted as a function of the following 9 variables:

$SAT = 761.54 + 0.0061(pctfree) - 1.73(pctblack) + 6.32(pctother) + 0.12(adm12) + 0.89(pctsrsat) + 0.38(pctmast) + 0.22(pctsydoc) - 1.90(avgexp) + 4.73(avgsal)$

where the variables are as follows (with their correlation coefficients and significance levels respectively listed in parentheses):

(pctfree) Percentage of Students in Free or Reduced Lunch Program (0.171; 0.009);

(pctblack) Percentage of Students African American (0.762; 0.000);

(pctother) Percentage of Students of Other Races (0.432; 0.000);

(ADM12) Average Daily Membership Grade 12 (0.495; 0.000);

(pctsrsat) Percentage of Seniors Completed SAT (0.494 0.000);

(pctmast) Percentage of Teachers with Masters degree(0.250; 0.000;

(pctsydoc) Percentage of Teachers with Six Year Certificate or Doctorate (0.288; 0.000);

(avgexp) Average Teachers' Years of Experience (0.147; 0.022);

(avgsal) Average Teachers' Salary (0.492; 0.000) (one tailed significance here), with $F = 49.20$, significance $= 0.000$ and multiple regression coefficient square $r^2 = 0.713$ and standard error of estimate $= 46.50$.

Questions of sparsity of the models and validity of tests cannot be addressed here. The author however believes that it is important to address the problem from the point of view of lesser complexity and build upon it by adding and studying new factors.

It has been noted that the number of students who take SAT increases with incomes in their families and so do their scores; it is thus argued, that, since the increasing population (sample) of takers reduces the mean score, comparisons among different states or schools may not make sense (Powell & Steelman, [12, 13]).

Individual and school background characteristics are strong predictors of SATs, and together account for over one fifth of their variance in (for instance) the California public school SAT-taking population (Rothstein, [14]). These models indicate that characteristics of students' schools, though not individual race and gender, account for a large share of the SAT's predictive power. Furthermore (ibid.), in sparse models the SAT may serve as a proxy for student background characteristics as they account for a big share of the variance in SAT scores. They are also strong predictors of freshmen GPA, in that they, together with high school GPA, school and individual demographic variables explain 45% of the variance in freshmen GPA, approximately the same as do SAT and high school GPA, in models which exclude background variables. The SAT score captures background characteristics more than it independently measures student preparedness (ibid.). On the basis of data from the University of California, (Rothstein [15]) finds that SAT scores are themselves highly correlated with family background. Racial composition of schools is very important and forgotten in a typical analysis and that is why SAT is overstated as a predictor of college performance in various studies.

Several prominent colleges have deemphasized importance of SAT scores in their admissions (notably University of California among them). Not only is SAT I deemphasized, but subject specific SAT II as well (Geiser & Studley [9]).

## 2. SAT at TAMUG

This is a study of placement in calculus classes at a specific institution, namely Texas A&M University, Galveston branch campus (TAMUG), in the duration of 5 semesters.[1] The data available to the author shows that, less than half of students in Calculus I classes had taken either SAT or ACT. Since, in this population, the number of ACT takers is a small subset of students who also took SAT (alas one student took ACT, not SAT), we will concentrate only on SAT here. The percentage of observed students who did take SAT was 41.56% (the number of high school graduates who take SAT in Texas and nationally oscillates around 50%) The mean scores for math SAT of white students (and TAMUG students are predominantly white) was 536 (Texas) and 532 nationally, for 2007 (all students mean math score was 507 for Texas and 515 (509 for public schools) for all schools nationally.)

The students who did take the tests usually enrolled earlier while those who did not take the tests were enrolling as late as the first class meetings. Some small colleges (TAMUG has less than 2000 students) often have problems recruiting students, sometimes due to student population decline in the area, lack of imaginative and aggressive or competitive recruitment aptitude by the college administration, etc. This is one of the reasons that colleges often relax all standards in recruitment in order to get the enrollment figures up and many students are admitted with no requirements of predefined academic achievement.[2]

TAMUG apparently advised at least some students as to what mathematics courses they should take when they entered college. These suggestions were not

[1] While this is a case study, the author believes that the questions raised and results obtained here should be valuable to any teacher, mutatis mutandis, irrespective of educational system he operates within. The subject of diagnostic testing is not isolated from numerous issues related to the complex area of mathematics education. Due to space limitations, comments related to the wider issues or causal relationships may at best be relegated to footnotes in this paper.

[2] Increasingly, colleges and universities are structured as corporations both in their hierarchies as well as in their profit motivated goals and actions. The high enrollments ultimately translate into higher cash flow from respective states, federal government, gifts and endowments (and partly from students' tuitions that are continuously and substantially raising). Universities (and in these matters it is the administrators and/or politicians, not the faculty, that have the exclusive say) find any idea of raising quality of education for its graduates patently odious, if such ideas for strengthening the intellectual level of its students would even slightly reduce the number of graduates or students enrolled. The "defenders" of the corporatization (and debilitating trivialization) of American educational system state as their main argument that "unlike other educational systems, education in America is not only for the select few, but for everybody ... " Regardless of whether this is in fact true, this noble goal however rests crucially on the meaning of the word "education," which, in a corporate, for profit, educational system means no more and no less than acquisition of a diploma at the end of this conveyor belt educational assembly line.

compulsory and students ignored them as they pleased, based on their own perception of their preparation and expediency in "getting through college faster." The suggestions were made on the basis of numerical value obtained by adding half of student's SAT score (or appropriately scaled ACT score) (max 400) and the departmental algebra test score (max 200) and trigonometry test score (max 200); if a student did not take SAT (ACT), then algebra and trigonometry scores were multiplied by two (Dimitrić, [6]). SAT, ACT or departmental tests are called *the pretests* here; there were a number of students who did not take any pretests. A number of students did not have any of these scores to go by and their placement was a makeshift, last minute decision.

This author had SAT scores (for the mathematics part, i.e. these scores halved, to be precise) for all the students who took SAT and were enrolled in his calculus classes.

## 3. Models used

Most frequently used models concerning college readiness $y_i^*$ of a student $i$ are in the form

$$E[y_i^*|X_i, SAT_i] = \alpha + \beta X_i + \gamma SAT_i, \tag{1}$$

where $SAT_i$ is the student $i$'s SAT score (we mean composite, not only math), $X_i$ is a vector of other admissions variables, and $y_i^*$ is a measure of the student's preparedness for college. Then, given observations on a realization $y$ of $y^*$, for a random sample of students, best linear predictor coefficients $\alpha, \beta$ and $\gamma$ can be estimated by OLS regression of $y$ on $X$ and $SAT$. Predictive accuracy is measured by the regression goodness-of-fit tests. More often than not, in SAT validity studies, $X$ is the high school GPA or class rank, whereas $y$ is the freshman GPA (Rothstein, [14]).

These studies generally measure the importance of SAT by either fitting the restricted model where $\beta = 0$ and the increment to fit conferred by the unrestricted model (1) over the model for which $\gamma = 0$. Thus, in addition to (1) the following regression models are utilized

$$E[y_i^*|X_i] = \alpha_2 + \beta_2 X_i \tag{2}$$

and

$$E[y_i^*|S_i] = \alpha_3 + \gamma_3 SAT_i \tag{3}$$

I will follow the third model, namely that of the extent of (linear) relationship between student SAT scores and their performance in class, mainly due to limited data that was available to me. In fact SAT scores will be paired with the first (midterm) test students took in my classes, then with their overall (cumulative) class score. The latter is a weighted mean of students' standardized scores throughout the calculus course they took at this time, as follows: $0.15 \cdot \sum_{i=1}^{3}(T_i - \mu_{T_i})/\sigma_{T_i} + 0.05 \cdot \sum_{i=1}^{3}(V_i - \mu_{V_i})/\sigma_{V_i} + 0.4 \cdot (F - \mu_F)/\sigma_F$, where $T_i$ stand for three midterm test scores (each weighted 0.15), $V_i$ are scores of homework, quizzes and in class activity respectively, each weighted by 0.05 and the final exam score $F$ weighted with 0.4; $\mu$ and $\sigma$ stand for the mean and standard deviation of the corresponding variables.

The first midterm test covers material that students should have ideally learned before they come to college (but they most likely have not): Domain, codomain and range of a function, composition of functions, vectors in 2D and the dot product, some simple limits ... At the point of the first test students have not gone through the whole course and it is likely that they are somewhat shocked to realize that what they have done in high school differs considerably from what they are doing now, primarily in that the demands on them are so much greater than in their pre-college years. Their cumulative score captures the short time-series of their work throughout the course, with the weight given greatly to the final exam (which is comprehensive).

I will look now into specific pooled data of two identical Calculus I classes and again, only for the mathematics score on the SAT, not the composite score (from fall 2002).

In class A[3] two sections were pooled together since they had identical conditions (same book, same teacher, were taught on the same days of the week and had same tests (up to minor variations). The first striking problem in using SAT for predictive purposes was that only 32 students had reported (or taken) SAT test and 45 did not (i.e. 41.56%; this is somewhat fewer than the percentage of SAT takers which nationally was about 50%.

Data shows (see data appendix) that the initial pool (at the time of the first mid-term exam) consisted of 32 students who took SAT and 45 who did not. The students who took ACT were a subset of the set of students who took SAT (with one exception of a student who took ACT, but not SAT). Number of ACT takers was 15 and for these reasons I omit looking here into the ACT, while awaiting for a richer data set. The SAT takers had a mean score on the first calculus test of $\mu(A2) = 46.59/100$ with standard deviation of $\sigma(A2) = 25.60$, while non-pretest takers had the mean $\mu(A4) = 23.5778, \sigma(A4) = 14.6217$ – about half of $A2$, which is striking. At the end of the semester, the SAT takers had the overall class statistics: $\mu(A3) = 0.41$ and $\sigma(A3) = 0.84$ while the 19 of 45 non-takers who survived long enough to take the final exam had those statistics as follows: $\mu(A5) = -0.1779$ and $\sigma(A5) = 0.6177$[4] which is again remarkable for it shows that those who did not take SAT were even much less prepared than the SAT takers.

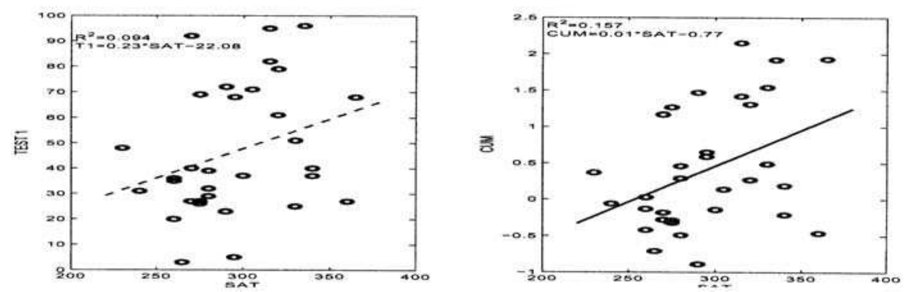
Let us now look into the extent of linear relationship between the SAT scores and the first test as well as the overall class score. One should note that, given

[3]Data sets are appended at the end of the paper, before the references, and for the benefit of the reader. In education circles thst treat mathematics education as a social science subject, data sets are often omitted from papers and instead placed at a designated websites, or simply claimed to exist. I shun that practice for at least two reasons: 1. Websites tend to be ephemeral and 2. I believe that it is important to be able to test the data for soundness and possible falsifications.

[4]The last score in this data set is clearly an outlier. It belonged to a student who started not that gloriously, as most of the students do not, and for that reason, was not pleased with the teacher. I persisted demanding of him to work harder and, since he was a mature, 'older' student, he did and he succeeded to get 91 on the final and an A in the course

a data set $x_i, i \in I$ and a linear relationship $y_i = ax_i + b$, then both data sets standardize to the same value, i.e. $(x_i - \mu_{x_i})/\sigma_{x_i} = (y_i - \mu_{y_i})/\sigma_{y_i}$; this is because $\mu_{y_i} = a\mu_{x_i} + b$ and $\sigma_{y_i} = a\sigma_{x_i}$. Note also that, for random variables $X, Y$ and constants $a, b, c, d$, $corr(aX + b, cY + d) = corr(X, Y)$. This is because $Cov(aX + b, cY + d) = acCov(X, Y)$ and $\sigma(aX + b) = a\sigma(X)$ and $\sigma(cY + d) = c\sigma(Y)$.

Here we find the correlation between the SAT score and the first test to be $corr(A1, A2) = 0.3058$ with explanatory power of SAT for the first test of 9.35%. Correlation between SAT and the cumulative class grade is $corr(A1, A3) = 0.3968$ with $r^2 = 15.74\%$. The scatterplots with least square lines are as follows:




Diag. 1. Linear regression SAT math scores v. First Test and Cumulative Class Score

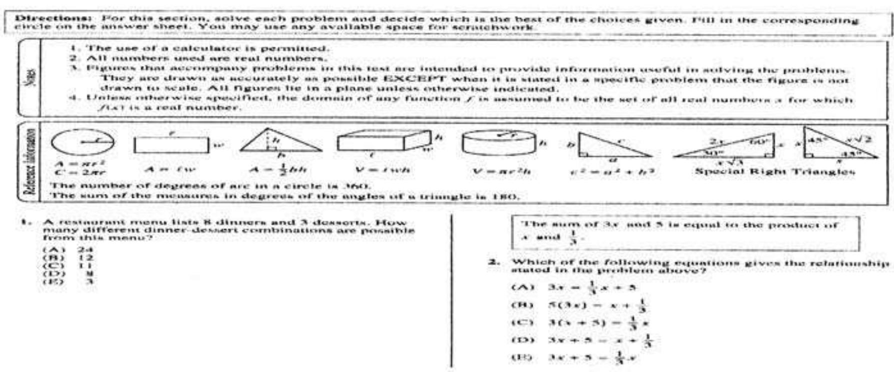
This low explanatory power of SAT math scores on my calculus students' performance is perhaps not unique. A University of California study (`http://www.universityofcalifornia/dotscommittees/boars/admissionstests.pdf`) found that SAT I scores only determined 7% of freshman year variance.

In the same study, SAT II scores (which test on more advanced material directly related to coursework) were a far better predictor (23% of variation). High school GPA accounted for 27% of variation.

The data of the same kind likely prompted Richard C. Atkinson, the then president of the University of California, to urge the elimination of the SAT as a requirement for admission. (The tipping point in his "epiphany" may have come after a visit to the upscale private school his grandchildren attended. There, he watched as 12-year-olds were drilled on verbal analogies, part of an extended training that, he said in announcing his proposal, "was not aimed at developing the students' reading and writing abilities but rather their test-taking skills." More broadly, he argued, "America's overemphasis on the SAT is compromising our educational system.")

Indeed the emphasis on preparations for taking these tests and gaining higher scores is one of the factors responsible for students being short on learning and developing mathematical thinking abilities, which must be the most important part of mathematics education. There are coaches who can get you well prepared and exercised in SAT taking and the SAT industry seems to be as lucrative as any of the test variety industries. Nationally, there are more than 50% of students who take SAT at least twice, which seems to improve their scores. Typically, students who take the test a second time see a 30-point increase on their combined score. Furthermore, students who take PSAT (preliminary SAT) score on average 60 points higher than those who do not. This dependence on test drilling comes to light when there are (even small) changes to the test, and this is when the test scores drop, for the prep industry has not had time to retool for the "new" test.

Similarly, SAT low explanatory power on my introductory calculus classes performance may be attributed to several factors: a) The time gap between student SAT taking and their first calculus test may be several months or a year. That period is too long for a SAT test-taking ability to hold and the students must have forgotten everything by the time of their attendance in their first college class. b) SAT I does not test trigonometry or any slightly advanced topics (in fact, I have mentioned that most of the test problems are at the level of not exceeding the 8th grade); on the other hand my calculus courses assume knowledge of trigonometry and that topic is not used as a "bonus" or "advanced," rather as an integral part of the course; trigonometric functions are functions like any other functions (if not "more important" than the rest).[5] c) Calculators (including graphing) have been admitted during SAT taking since 1994, while they are not allowed in my calculus tests.[6]. d) The following excerpt from a model SAT test shows that the students are not assumed to know (for purposes of taking the test) how to calculate areas of circles, or for that matter triangles and rectangles, whereas I assume this to be part of a required basic numeracy for students taking calculus.



**Directions:** For this section, solve each problem and decide which is the best of the choices given. Fill in the corresponding circle on the answer sheet. You may use any available space for scratchwork.

Notes

1. The use of a calculator is permitted.
2. All numbers used are real numbers.
3. Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that the figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.
4. Unless otherwise specified, the domain of any function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.

Reference Information

$A = \pi r^2$, $C = 2\pi r$ $\quad A = \ell w$ $\quad A = \frac{1}{2}bh$ $\quad V = \ell wh$ $\quad V = \pi r^2 h$ $\quad c^2 = a^2 + b^2$ $\quad$ Special Right Triangles

The number of degrees of arc in a circle is 360.
The sum of the measures in degrees of the angles of a triangle is 180.

1. A restaurant menu lists 8 dinners and 3 desserts. How many different dinner-dessert combinations are possible from this menu?
(A) 24
(B) 12
(C) 11
(D) 8
(E) 3

The sum of $3x$ and 5 is equal to the product of $x$ and $\frac{1}{3}$.

2. Which of the following equations gives the relationship stated in the problem above?
(A) $3x = \frac{1}{3}x + 5$
(B) $5(3x) = x + \frac{1}{3}$
(C) $3(x + 5) = \frac{1}{3}x$
(D) $3x + 5 = x + \frac{1}{3}$
(E) $3x + 5 = \frac{1}{3}x$

[5] Some more selective colleges rely on SAT II scores and "advanced placement" (AP) tests to ensure their students are academically prepared for college.

[6] This may be a helping factor; there was a short period when I allowed students to use calculators, but I noticed that their "use" was rather superficial and of the "false-security" kind that seemed to waste their time and keep them away from applying their brains. Students' performance was in fact slightly better when they did not use calculators

### 4. Examples of deficiencies

The recurring problem showed itself yet again in that most students come unprepared for the class (80-90% are referred to for remediation with tutorial services at the College).[7] There are many things that students need to be "updated" with, but some of them are crucial to students' further progress in calculus. What are these problems?

"I have observed oftentimes that the greatest part of the difficulties the mathematical laborers stumble upon in learning the Analysis of the infinite stem from the fact that they hardly understand basic algebra while appealing to this more sublime art; therefore, not only that they remain at the sideline but they form perverse ideas about infinity, the very notion crucial to the vocation. For learning the Analysis of the infinite, neither complete knowledge of basic algebra nor all the art invented thus far are needed; while there are many questions whose reading will enable for deeper preparation for this sublime science, elements of basic algebra are either omitted or are not treated thoroughly [in treatises on the subject]."

These are not the words of the author of this paper but of the great Leonhard Euler, spoken 250 years ago (Euler, [7])![8]

Deficiencies in students' background necessary for a successful understanding of a beginning calculus course are numerous and diverse to the extent that can overwhelm an instructor who dwells upon them. Although the extent of these deficiencies varies with individual students, there are some that almost all students have. Such are almost universal inability of students to perform operations with fractions, especially if the fractions contain variables or constants not given as concrete numerical values.[9]

---

[7] Most colleges nowadays have remedial/tutorial services and at some places these are big departments with their own budgets, etc. Remedial courses or tutoring are often taught by teachers who are not well prepared, or students who may need the very remediation services they are presumably servicing. There are no serious studies showing usefulness of these remedial courses for students' mathematical well-being. However colleges have fully embraced remedial courses because they generate a substantial cash flow. In a recent trend, some states see remediation as somewhat of an embarrassment of their precollege educational systems and/or as waste of college resources; for that reason states, such as Ohio, etc. have decided to cut funding of courses that have even a hint of remediation in their name. The colleges responded promptly (lest they lose substantial funding) by changing names of the remediation courses to sound more like college level courses. Thus a course previously named "A prep pre-algebra" may be renamed "Chosen topics in foundations of calculus . . . "

[8] This is the author's translation of the relevant part of Euler's original preface. Like most 18th century texts in Latin, this one is difficult for accurate translation. The Latin original of the passage is as follows: *Sæpenumero animadverti, maximam difficultatum partem, quas Matheseos cultores in addiscenda Analysi infinitorum offendere solent, inde oriri, quod, Algebra communi vix apprehensa, animum ad illam sublimiorem artem appellant; quo fit, ut non solum quasi in limine subsistant, sed etiam perversas ideas illius infiniti, cujus notio in sublsidium vocatur, sibi forment. Quanquam autem Analysis infinitorum non perfectam Algebræ communis, omniumque artificiorum adhuc inventorum cognitionem requirit; tamen plurimæ extant quæstiones, quarum evolutio discentium animos ad sublimiorem scientiam præparare valet, quæ tamen in communibus Algebræ elementis, vel omittuntur, vel non fatis accurate pertractantur.*

[9] This problem of deficiencies in elementary mathematics background has been well recognized. In recent times one "solution" to this problem came in the guise of the so-called "calculus

Let us see how this manifests itself in a typical sample taken from my calculus 1 tests.

**A test problem 1.** Find the derivative function of $f$, using appropriate differentiation rules. For every step state the rule you are using ($a$ is a given constant):

$$f(x) = \frac{-2ax + \frac{3}{x}}{-4x}.$$

This was the time for students to show that they knew basic differentiation rules (no chain rule was covered at that stage yet). The students were told in class to distinguish the phrase "differentiation rules" from the phrase "find the derivative by a limit definition" and a number of examples were worked out of each kind.

Here is how one student starts "solving" the problem:

$$\lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{x\to a} \frac{\frac{-2ax+\frac{3}{x}}{-4x} - a}{x - a}.$$

He did remember a limit definition of the derivative of $f$ at a point $a$, (and this is not what was asked for in the problem), but he is "unfortunate" in that there is a constant $a$ in the definition of the function, which is the same letter used in the generic limit definition of the derivative at $a$ that he remembers. Even worse, he "forgets" to use expression for $f(a)$, and simply uses $a$ instead. Thus, at the start there are several problems that are driving the student nowhere:

(i) He is not following instructions carefully and/or does not remember to distinguish between finding the first derivative by (a limit) definition and by applying the differentiation rules. This points to several background problems, such as poor mathematical culture, but more often than not, poor command of English.[10] Consequently, command of English, in fact, does play a role in students' performance on mathematical tests, but that command as well is not best assessed through conventional language tests; this prompts a thought of trying to use the combined SAT scores (not solely math scores) in order to establish their explanatory value in performance variation.

(ii) Constants when used in form of letters (rather than concrete numeric values) are often a stumbling block to students. This comes from problems with

reform" that professed to place emphasis on teaching mathematics by "conceptual understanding." Further analysis of this noble concept shows that the underlying "philosophy" was to in fact curtail as much elementary mathematics, such as simple algebra and arithmetic, from teaching, textbooks or examinations. If students do not understand the essentials about the quadratic functions and equations, then the "reformed calculus" would attempt to find ways to avoid using such troublesome constructs!

[10] Contrary to the conventional wisdom, the culprits here are mostly native speakers. While foreign born students who are fresh immigrants often speak hardly understood English, they, perhaps unexpectedly, often have better command of mathematical English (in a written form, including when reading/writing tests) than the native speakers. This may be explained by the fact that they have better grasp of mathematical language they have acquired in high schools in their countries and that mathematical language is often structured similarly in its logical constructs within different natural languages.

abstraction (which is the single most essential component of the core of mathematics). Given a concrete formula for a function $f$ in the form $y = f(x)$, a student may be able to write what $f(1)$ is (but not all students can), especially if the formula for $f$ does not contain any constants denoted by a letter (rather than its concrete numerical value). The difficulty arises quickly when a student has to write what $f(b)$ is for an unspecified constant $b$, let alone if $b$ is an algebraic expression that is "confusing". Again this kind of a problem stems from absence of a general mathematical culture.

(iii) Lack of basic ability to perform elementary operations with fractions. Thus the same student "solving" the problem in question continues as follows:

$$\lim_{x\to a} \frac{\frac{\frac{-2ax^2+3}{x}}{-4x} - \cancel{a}}{x - \cancel{a}} = \lim_{x\to a} \frac{\frac{\frac{-2ax^{\cancel{2}}+3}{\cancel{x}}}{\cancel{4}x}}{\cancel{x}} = \lim_{x\to a} \frac{-2ax+3}{3x} = \lim_{x\to a} \frac{-2a+3}{3}$$

Inability to perform operations with fractions is most widespread and among them cancellations of summands in numerator and denominator predominate. Not infrequently, multiplicative factors are canceled by subtractions of the base (as this student does) rather than subtraction of the exponents.

Taking exponentiation as an additive operation is another perpetual problem with many students. A typical representative of this kind of errors may be seen from a "solution" given by a student to the following problem:

**A test problem 2.** State precisely the intermediate value theorem, then use it to prove that the following equation has at least one real root. Locate that root between two whole numbers: $\sqrt{x-16} = 1/(x-5)$

The "solution" runs as follows:

$$\sqrt{x} - \sqrt{16} = 1/(x-5), \quad \sqrt{x} = (x-5)^{-1} + 4$$

Aside from the fact that the intermediate value theorem is devalued (!) by an attempt to explicitly solve the equation, we see a rather frequently invoked "additivity" of square root and likewise additivity of the reciprocal in continuation of the "solution:" $\sqrt{x} = x^{-1} - 5^{-1} + 4, \quad x^{1/2} - x^{-1} = -1/5 + 4$, "Answer: Between 4 and 5" (never mind that this "solution" is not even in the domain of the root in the starting equation).

## 5. A diagnostic test

This author clearly could not rely on the canned test scores (such as SAT) to be his crystal ball to tell him the extent of students' preparation and likelihood of success in a beginning calculus class. With the lack of adequate gages a diagnostic test had to be constructed. The basic source for shaping questions for such a diagnostic test were concrete test solutions of students from previous calculus tests (midterms and finals). The author chose to test for a few areas taken from a sample of most characteristic deficiencies in algebra and trigonometry deemed to be an absolute minimum of required background for a beginning calculus course. Choosing different samples from this deficiency pool would result in different diagnostic tests.

This diagnostic test is given at the beginning of every calculus 1 course, at the first class meeting. It lasts for 30 minutes of the class and calculators are not used. The students are told that it is not a part of their class grade but that they should make an effort as if it were. They dutifully take the test applying themselves to it. Following is the test I use most of the time (with my short comments in brackets).

**Intro-Quiz for Calculus I**

1) Mark the $y$-axis in the following rectangular coordinate system:

[a diagram of the unmarked, oriented perpendicular axes was placed here; unappetizingly, a number of students is unsure what the usual markings of the axes are]

2) Sketch the curve $y = -2x - 1$.

[a diagram of the unmarked, oriented perpendicular axes was placed here; sketching (or anything else about) a straight line should be taken for granted. But an inordinate number of students have problems with it, or if they eventually manage it, it is after considerable struggle, rather than automatic, as it should be.]

3) Find the intersection(s) of the curves $y = x + 1$ and $x = -y + 1$.

[With very integer (preferably positive and small) coefficients students somehow guess the answer, anything seemingly more complicated causes headaches. This is a reasonable trouble, for it has $y$ as a function of $x$ in the first equation, and the other way around in the second equation]

4) Sketch the curve $y = -x^2 - 1$.

[a diagram of the unmarked, oriented perpendicular axes was placed here; quadratic equations and parabolas are crucial, but they turn out to be hard]

5) What are coordinates of the tip (or bottom) of the curve $y = -x^2 - 1$?

[If a student sketched this successfully, they would eventually find the tip; if not it usually proved to be a hard problem]

6) Find algebraically the points of intersection of the two curves $y = -2x - 1$ and $y = -x^2 - 1$.

[Relationship between curves of two lowest degrees is of utmost importance]

7) What are the roots of the equation $x^2 - 2x - 1 = 0$?

[Again one finds that students solve quadratic equations by guessing, or trial and error. Many have in fact forgotten the quadratic formula and this is why there is a great failure rate in solving this equation]

8) What are all the $x$ between 0 and $2\pi$ such that $\sin x > \cos x$?

[Calculus should be supported by pillars of variety of functions. Trigonometric functions are indispensable and so are basic manipulations with them]

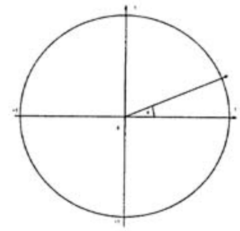
9) Compute $\sin^2(78.5°) + \cos^2(78.5°) =$

[A surprising number of students have difficulty with the Pythagoras' theorem and its trigonometric form in particular]

10) Can you simplify $\frac{\tan 5x}{5} =$

[Temptation is great and perceived multiplicativity (or linearity) of trig functions is often a stumbling block in exercises that have trigonometric functions]

11) Can you simplify $\cos 2x \cdot \tan 2x =$



[Knowing how tan is defined via sin and cos is certainly a must]

12) Given the unit circle in the diagram, and an angle $x$. Mark where you would read $\sin x$ and $\cos x$.

[Knowing the unit circle is the key to quick grasp of trigonometry.]

13) Using inequality symbols, how would you write that a quantity $x$ is not less than -1 and is less than or equal to 5?

[When finding a domain of a function, inequalities will certainly pop up, not to speak of finding $x$ for which the derivatives are positive. Students stumble here mightily, anywhere from not knowing the symbols for greater or smaller (which is which) to not knowing that inequalities switch after multiplication by a negative number, etc. ]

14) Shade on the x-axis all $x$ for which the inequality $|x-2| > 1$ holds.

[Importance of ability to work with the absolute value function cannot be overemphasized. While students can eventually struggle out absolute values of concrete numerical expressions, functions defined by absolute values present great difficulty]

15) Factor $a^2 - b^2 =$

[This expression appears all the time, but not all students can factor it. ]

16) Expand $(a+b)^2 =$

[Those who solve it seem to be doing it by multiplication (a+b)(a+b); the answers are often along the line of additivity of the square function, or other "interesting mathematical inventions."]

17) Can you simplify $\sqrt{a^2+b^2} =$

[The temptation is too great and rather large number of students assume linearity of square roots in their "solutions" of calculus problems.]

18) Convert into one fraction:

$$\frac{5+x}{7+y} - \frac{x}{y} =$$

[Can they add fractions in an abstract form?]

19) Convert into one fraction:

$$1 + \frac{1}{x} =$$

[Adding fractions and non-fractions is a must, but many students would give $2/x$ as an answer.]

20) Simplify so you only have positive exponents:

$$\frac{(2y)^{-2}}{y^2} =$$

[Rudimentary exponential rules are a must to be able to play with the exponentials]

21) Write as an exponent of $a$:

$$\sqrt[3]{\frac{1}{a^2}} =$$

[This is harder than one would hope]

22) $7 - 2 \times 4 =$

[The conventional precedence of operations does count, but some students do not know it. Could it be attributed to calculator use?]

23) Calculate

$$\frac{3}{\frac{1}{3}} =$$

[Double fractions are fractions, like any other. Many students do get this one, but some do not. The number of solvers would change if the numerical values were to be replaced by letters.]

24) Compute $-2 \cdot (5 - 5 \cdot 3 + 1) =$

[Parentheses, order of operations, negative multiples, all show in concrete exercises, but a variety of answers here point to great difficulties for the students]

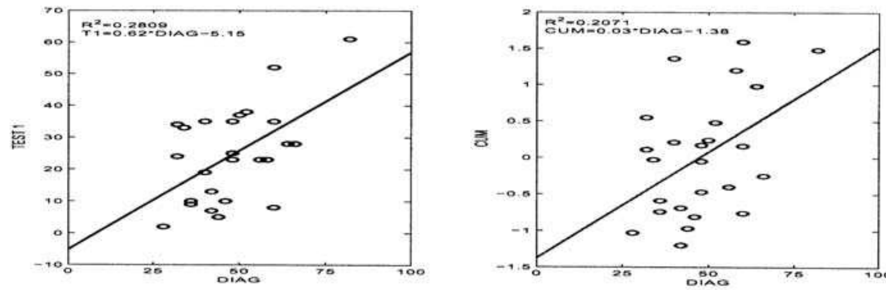
25) Calculate $(1/2)^3 =$

[Exponentials of fractions are important; many do get this one, but 1/6 as an answer does also show]

## 6. The diagnostic test data and discussion

Taking information from the data section, we see that the explanatory value of the diagnostic test scores (DIAG) on the first test scores taken in class (TEST1) for classes B, C, D, E is respectively $R_T^2 = 0.2809, 0.256, 0.0799, 0.2369$ with mean explanatory portion of the diagnostic test on the first test being $R_{DT}^2 = 0.2134$. The set of squared correlations of the diagnostic test scores CUM with cumulative class scores is as follows: $R_C^2 = 0.2071, 0.1971, 0.2757, 0.3018$ with mean explanatory part of the diagnostic test scores on the final class score $R_{DC}^2 = 0.2454$.

The scatter-plot for data in class B is as follows:




Diag. 2. Linear regression DIAG math scores v. First Test and Cumulative Class Score

Thus, our predictive role of the diagnostic test on the overall class score is close to $1/4 = 25\%$. Can there be tests with better predictive power? We might dare say that, with fine tuning of the test, and its expansion with more questions and doubling its workout duration to 50 minutes, we may strengthen its explanatory power on the class scores to $1/3$. In a social science, like education, it is a high value. It would be unrealistic to expect higher values because student preparation is not the only educational component (cf. Dimitric [5]), rather one of the following four (that we believe to be most important): 1) Adequacy of students' background for a particular course, 2) Amount of students' work (efforts) in a particular course, 3) Standards in educating, examining and evaluating, 4) Quality of teaching in a course. The numerical statistics are conditional, namely, given other educational components, we can estimate how much students' math background will influence their performance in a (beginning) calculus course. It would not be realistic to expect that the mean explanatory power of a diagnostic test is higher than $1/3$, for it would sound as if that component is almost deterministic, where other components (such as teaching, students industriousness, etc.) would appear to have no great impact.

The results obtained here are compatible with a similar study in relation to statistics classes (Dimitric [5]). There, the mean $R^2$ was 0.32, but the range (and deviation) was considerably greater since, unlike here, classes considered were at different institutions with different kinds of student populations (from heterogeneous to homogeneous). It is not surprising that the diagnostic tests for calculus and statistics courses have large overlap – diagnostic testing of arithmetic and elementary algebra skills is a must, and so is some prodding of a general mathematical culture of the students.

Is this test more reliable as a (linear) predictor of student performance? Our test seems to be of greater explanatory value for subsequent student performance than the SAT and we have already itemized some reasons for this fact. We thus believe that the "home made tests" like ours (if carefully made with no drilling or preparations for them) are a better tool than the canned SAT (and our limited data does not give us reason for any different conclusion regarding the ACT).

We believe that the canned test scores can be wholly replaced by the scores from in-house diagnostic tests in assessing likelihood of students' success in a particular class. However, if the canned test data are available, they should not be completely ignored. This is especially important in situations where some students are admitted without having taken any of the canned tests. As shown above, the mean test (or class) score of students who did not take any of the canned tests are considerably lower (standard deviation is lower too) than those who did (cf. data for A2, A3 and A4, A5) and that is an important information to have.

There is another useful statistic that can be drawn from the diagnostic test scores; it is the lowest diagnostic score (call it cutoff scores – $coffs$) among the students who passed the class. $Coffs$ for classes B, C, D, E were respectively 32, 44, 32, 34 (mean 35.5).

The diagnostic test data is a tool that can be utilized promptly, at the start of the class. Students can be advised of their strengths and deficiencies. The questions in the diagnostic test are rather easy in comparison to what rigor and demands await the students in the first calculus course. A student is well-prepared if his diagnostic test score is 90 and above (it rarely happens). Students whose diagnostic tests are below 32 can be safely advised to drop the course before they start it and those in the range of 33 to 85 should seek help. This will result in large portion of student body being sent to remediation (in an ever increasing roles colleges play as surrogates for high and elementary schools). The evidence and actions the diagnostic tests will call for, may or may not get a sympathetic ear, depending on the state of administrators' minds at any particular institution (Dimitric [6]). One must make sure to remember that, although the correlation between the diagnostic test scores and the final class scores is high (about 50%) the diagnostic score does not offer a deterministic cause of student's grade.

Finally, this is a case study, with a limited pool of data the inferences were made from. It should serve as a starting point for a more thorough study with greater data pool and consideration of greater variety of factors influencing student outcomes.

## 7. Blooms taxonomy and my test

A broad view of taxonomy of educational objectives (Bloom's taxonomy) regarding measurable student outcomes consists of cognitive objectives (knowledge-based goals), psycho-motor objectives (skill-based goals) and affective (values, attitudes, and interests) objectives (Bloom [3]); see also a revised version in (Anderson et al. [1]). Each of these components has its own taxonomy. Thus knowledge-based taxonomy consists of (in increasing level of sophistication or complexity): knowledge (remembering), understanding, applying, analyzing, evaluating and creating (the latter three are often at the same, top level of the cognitive tree). One does not have any insight into these aspects of students' performance from SAT scores alone. In this light, we can add to our reasons why SAT scores have low predictive value of students' performance in my calculus classes: SAT tests at the cognitive tree level are below the level of the course tests (midterms and finals). My diagnostic tests are more informative for I have concrete feedback after the students take the diagnostic. The diagnostic does measure students' knowledge, understanding and applying, but in a small way measures a level of students' mathematical culture (mostly non-existent) when they come to college. Thus knowing that $\sqrt{a^2+b^2}$ does not equal to $a+b$ is not as much the matter of knowledge or memorizing, rather a fundamental understanding that there are functions that are not linear and that the square root function is one of them. We all know that learning "Salsa" (or any other dance) includes learning the choreography; and more importantly, the defining rhythm. This "rhythm" is the mathematical culture for us. It can be safely assumed that students who are poor at the lower levels of the cognitive tree, are most likely to be even poorer in the top level of the taxonomy. Likewise, analysis of student errors that are used to construct the diagnostic test shows ubiquitous problems already at lower levels of the cognitive tree, and thus the role of the diagnostic test is in the realm of the basic question: What students are unprepared in the realms of the lowest levels of the taxonomy. Rare excelling students are quickly identified, but because our educational system mostly caters to students below par, the well-performing students must be given attention they deserve, through out-of-class and special assignments.

The questions related to Blooms taxonomy are relevant here, but their exhaustive treatment in the context of our tests would warranty a long paper, or perhaps a book.

Acknowledgment. The author gratefully thanks Professor François Pluvinage for a constructive critique of this paper at ICME11 and in particular to his prompting the author's attention to the question of Bloom's taxonomy. Thanks are also due to a number of referees who, through their comments, contributed to this paper being less imperfect than when it was first written.

## 8. DATA

The range of the class tests and my diagnostic test is always from 0 to 100.

**Class A:**

($n=32$) students had SAT scores and $n=45$ students did not take SAT/ACT; the SAT scores of these 41.56% of the students who took SAT is as follows:[11]

A1=[260 320 340 230 275 335 290 315 360 330 270 315 270 305 275 280 300 275 295 290 280 265 240 260 340 330 260 280 295 270 320 365];

with mean $\mu(A1) = 294.8438$ and standard deviation $\sigma(A1) = 33.6127$.

The scores of these students on the first test were:

A2=[35 61 40 48 69 96 72 95 27 51 27 82 92 71 26 32 37 27 68 23 29 3 31 36 37 25 20 39 5 40 79 68];

[11] These are apparently half of students' SAT math scores. I was told that the department had an internal test in algebra and trigonometry (given to at least some students) that was used, in conjunction with SAT, to place students (by way of advisory) in math classes. (Dimitric [6])

with $\mu(A2) = 46.5938, \sigma(A2) = 25.5997, corr(A1, A2) = 0.3058, R^2 = 0.0935$ and the least square prediction line $\hat{A}2 = 0.23 * A1 - 22.08$

The overall class score

A3=[-0.13 0.27 0.19 0.37 1.27 1.92 1.47 2.15 -0.46 0.49 -0.28 1.42 1.17 0.14 -0.32 0.46 -0.14 -0.29 0.65 -0.89 -0.49 -0.71 -0.06 0.03 -0.21 1.54 -0.42 0.29 0.59 -0.18 1.31 1.93];

with $\mu(A3) = 0.4087, \sigma(A3) = 0.8388$, $corr(A1, A3) = 0.3968, R^2 = 0.15745$ and the least square prediction line $\hat{A}3 = 0.01 * A1 - 0.77$

Total of 45 students did not have SAT (or ACT) scores. Their first test scores were

A4=[12 14 23 19 14 32 50 44 63 33 39 50 27 28 20 50 25 2 7 20 3 46 12 8 7 28 23 15 19 16 29 20 16 27 47 4 31 17 33 19 19 25 2 8 15];

with $\mu(A4) = 23.5778, \sigma(A4) = 14.6217$

A subset of A4 who survived to take the final exam had the following overall class scores:

A5=[-0.55 -1.09 -0.38 0.52 0.35 -0.33 -0.2 0.33 -0.81 -0.14 -0.68 -0.52 -0.15 -0.27 -0.04 -0.5 -0.37 -0.33 1.78];

with $\mu = -0.1779, \sigma = 0.6177$. The last score here is an outlier (a student who had a score of 91 on the final) and if the outlier is removed, then $\mu = -0.2867, \sigma = 0.4064$.

**Class B:**

($n = 25$); the diagnostic test scores were as follows:

B1=[40 82 36 48 44 66 46 58 48 36 32 42 56 64 60 40 60 28 52 48 34 42 50 32 60];

$\mu(B1) = 48.1600, \sigma(B1) = 12.8572$.

The first class test scores were as follows:

B2=[35 61 10 23 5 28 10 23 25 9 34 13 23 28 52 19 8 2 38 35 33 7 37 24 35];

with $\mu(B2) = 24.6800, \sigma(B2) = 14.7753$, $corr(B1, B2) = 0.539$, $R^2 = 0.2809$; least square prediction line is $\hat{B}2 = 0.62 * B1 - 5.15$; the cumulative class scores were

B3=[1.36 1.48 -0.59 0.17 -0.97 -0.25 -0.81 1.2 -0.47 -0.74 0.11 -1.2 -0.4 0.98 1.59 0.21 -0.76 -1.03 0.48 -0.05 -0.03 -0.69 0.24 0.55 0.16];

with $\mu(B3) = 0.0216, \sigma(B3) = 0.8204$, $corr(B3, B1) = 0.4551, R^2 = 0.2071$, and least squares prediction line: $\hat{B}3 = 0.03 * B1 - 1.38$.

**Class C:**

($n = 17$) Diagnostic test:

C1=[84 64 54 56 68 68 44 20 56 64 58 52 60 78 10 52 34];

with $\mu(C1) = 54.2353, \sigma(C1) = 18.9193$; their first midterm test scores were

C2=[59 59 2 35 8 34 10 14 33 48 61 33 6 55 19 45 18];

$\mu(C2) = 31.7059, \sigma(C2) = 20.2971$; here $corr(C1, C2) = 0.5060, R^2 = 0.256$ and the least square linear predictor is $\hat{C}2 = 0.54 * C1 + 2.26$;

their final class scores were:

C3=[1.18 1.28 -1.22 0.2 -0.33 -0.8 0.4 -0.61 0.66 0.65 1.58 -0.19 -0.81 0.57 -0.55 0.09 -0.73];

with $\mu(C3) = 0.0806, \sigma(C3) = 0.8271$. Here $corr(C3, C1) = 0.4440, R^2 = 0.1971$ and the least square predictor line is $\hat{C3} = 0.02 * C1 - 0.97$.

**Class D:**

$(n = 14)$ Diagnostic

D1=[54 72 42 80 30 28 56 32 72 32 56 58 42 64];

with $\mu(D1) = 51.2857, \sigma(D1) = 17.2155$; the first midterm test

D2=[34 66 23 64 10 10 27 64 15 36 84 14 55 34];

with $\mu(D2) = 38.2857, \sigma(D2) = 24.1706$, $corr(D1, D2) = 0.2882, R^2 = 0.0799$ with least square predictor $\hat{D2} = 0.4 * D1 + 17.54$.

The total class score: D3=[-0.17 0.58 -1.01 0.57 -1.15 -1.01 -0.28 -0.25 0.37 0.29 1.81 -0.88 0.42 0.7];

with $\mu(D5) = -0.00071429, \sigma(D5) = 0.8381$, $corr(D1, D3) = 0.5251, R^2 = 0.2757$ with the least square linear prediction $\hat{D3} = 0.03 * D1 - 1.31$.

**Class E:**

Of the students who took the final exam, the diagnostic test scores were

E1=[52 82 70 48 28 90 62 72 34 76 38 48 48 38 50 52];

with $\mu(E1) = 55.5, \sigma(E1) = 18.0592$; the corresponding first midterm test scores

E2=[19 42 25 14 28 39 20 58 20 41 46 7 6 12 11 39];

with $\mu(E2) = 26.6875, \sigma(E2) = 15.6789$. $corr(E1, E2) = 0.4868, R^2 = 0.2369$ and least squares linear predictor: $\hat{E2} = 0.42 * E1 + 3.23$. Cumulative class scores were

E3=[-0.28 1.1 -0.05 -0.62 -0.8 0.95 1 0.56 0.06 0.76 1.2 -0.94 -1.41 -0.4 -1.29 0.16];

with $\mu(E3) = 0, \sigma(E3) = 0.8669$. Finally $corr(E1, E3) = 0.5494, R^2 = 0.3018$ and the least square linear predictor $\hat{E3} = 0.03 * E1 - 1.46$.

## REFERENCES


[1] Anderson, L.W.. Krathwohl, D.R., Airasian, P.W., Cruikshank, K.A., Mayer, R.E., Pintrich, P.R., Raths, J. & Wittrock, M.C. (Eds.), *A taxonomy for learning, teaching, and assessing – A revision of Bloom's taxonomy of educational objectives*, Addison Wesley Longman, Inc., 2001.

[2] Barro, R.J., *Economic viewpoint: Why colleges shouldn't dump the SAT*, Business Week, April 9, 20, 2001.

[3] Bloom, B.S. (Ed.), *Taxonomy of Educational Objectives: The Classification of Educational Goals*, pp. 201–207; Susan Fauer Company, Inc., 1956.

[4] Camara, W.J., *There is no mystery when it comes to the SAT I*, College Board News, 2001.

[5] Dimitric, R., *Components of successful education*, The Teaching of Mathematics, **6**, 2 (2003), 69–80. `http://elib.mi.sanu.ac.rs/files/journals/tm/11/tm621.pdf`. (See also ICME10 Congress publications: `http://www.icme-organisers.dk/tsg27/papers/09_Dimitric_fullpaper.pdf`).

[6] Dimitric, R., *Math education at TAMUG*, preprint, 2003.

[7] Euler, L., *Introductio in Analysin Infinitorum*, Lausanne: Apud Marcum - Michaelem Bousquet. [English translation by Blanton, J.D. (1988). *Introduction to Analysis of the Infinite*, Book I. New York, Berlin, Heidelberg, London, Paris, Tokyo: Springer-Verlag].

[8] Fox, S., Israel, E., O'Callaghan, R., et al., *The official SAT study guide: For the new SAT$^{TM}$ (For the March 2005 test and beyond)*, CollegeBoard SAT, New York: College Entrance Examination Board, 2004.

[9] Geiser, S. & Studely, R., *UC and the SAT: Predictive validity and differential impact of the SAT I and SAT II at the University of California*, manuscript, UC Office of the President.

[10] Hawkins, H. G., *Comparison of actual and predicted scholastic aptitude Test (SAT) scores. Accounting for the effects of racial composition, poverty, class size, and teacher characteristics*, 2001, South Carolina Jim Self Center on the Future, at http://www.strom.clemson.edu/publications/hawkins/SATperf.pdf, http://selfcenter.clemson.edu/filemgmt_data/files/SATperf.pdf.

[11] McWhorter, J.H., *Eliminating the SAT Could Derail the Progress of Minority Students*, Chronicle of Higher Education, March 9, B11-B12, 2001.

[12] Powell, B. & Steelman, L.C., *Variations in state SAT performance: Meaningful or misleading?* Harvard Educational Review, 54(4) (1984), 389–412.

[13] Powell, B. & Steelman, L.C., *Bewitched, bothered, and bewildering: The use and misuse of state SAT and ACT scores*, Harvard Educational Review, 66(1) (1996), 27–59.

[14] Rothstein, J. M., *College Performance Predictions and the SAT*, Journal of Econometrics, 121 (July-August) (2004), 297–317.

[15] Rothstein, J. M., *SAT scores, high schools, and collegiate performance predictions*, 2005. http://www.princeton.edu/~jrothst/workingpapers/rothstein_CBvolume.pdf.

DBRI, PO Box 382, PITTSBURGH, PA 15230-0382, USA

*E-mail*: dimitricr@member.ams.org